Chapter 16 Kinematics

1. A particle *P* moves in a straight line such that, *t* seconds after passing through a fixed point *O*, its acceleration, *a* ms^{-2}, is given by $a = -6$. When *t* = 0, the velocity of *P* is $18ms^{-1}$

   a. Find the time at which *P* comes to instantaneous rest.

   $v = 0$

   [3]

   $v = \int a\, dt$

   $= \int -6\, dt = -6t + c$

   $t = 0,\ v = 18$

   $18 = c$

   $v = -6t + 18$

   $0 = -6t + 18$

   $6t = 18$

   $t = 3s$

   b. Find the distance travelled by *P* in the 3rd second.

   [3]

   $s = \int v\, dt$

   $= \int -6t + 18\, dt$

   $= -\frac{6t^2}{2} + 18t + c$

   $= -3t^2 + 18t + c$

   $t = 0,\ s = 0$

   $c = 0 \quad \therefore s = -3t^2 + 18t$

   $t = 2,\ s = -12 + 36 = 24$

   $t = 3,\ s = -27 + 54 = 27$

   3^{rd} sec $= 27 - 24 = 3m$

2. (i) A particle *P* moves in a straight line such that its displacement, *x* m, from a fixed point *O* at time *t* s is given by $x = 10 sin\ 2t - 5$.

   a. Find the speed of *P* when $t = \pi$.

$v = 20\cos 2t$
$= 20$

speed $= 20$

[1]

   b. Find the value of *t* for which *P* is first at rest.

$v = 0$

$20\cos 2t = 0$

$\cos 2t = 0$

$2t = \cos^{-1}(0)$

$t = \frac{\pi}{4}$

[2]

   c. Find the acceleration of *P* when it is first at rest.

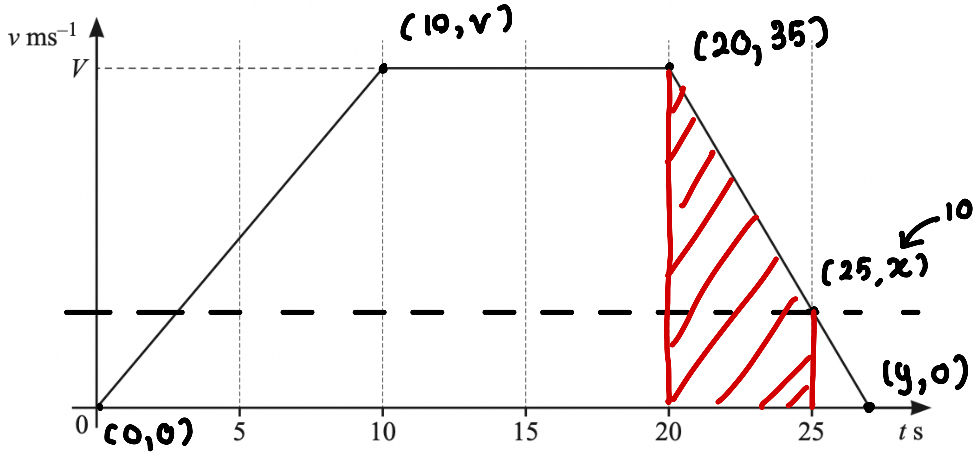
$a = -40\sin 2t$

$= -40 \sin \frac{\pi}{2}$

$= -40$

[2]

(ii)



The diagram shows the velocity–time graph for a particle Q travelling in a straight line with velocity $v\ ms^{-1}$ at time ts. The particle accelerates at $3.5ms^{-2}$ for the first 10s of its motion and then travels at constant velocity, $V\ ms^{-1}$, for 10s. The particle then decelerates at a constant rate and comes to rest. The distance travelled during the interval $20 \le t \le 25$ is 112.5 m.

a. Find the value of V. [1]

$$\frac{V-0}{10-0} = 3.5$$

$$V = 35\,ms^{-1}$$

b. Find the velocity of Q when t = 25. [3]

$$\frac{1}{2} \times (35+x) \times 5 = 112.5$$

$$35 + x = 45$$

$$x = 10$$

c. Find the value of t when Q comes to rest. [3]

$$m = \frac{10-35}{25-20} = -5\,ms^{-1}$$

$$\frac{0-10}{y-25} = -5$$

$$-10 = -5y + 125$$

$$5y = 135$$

$$y = 27$$

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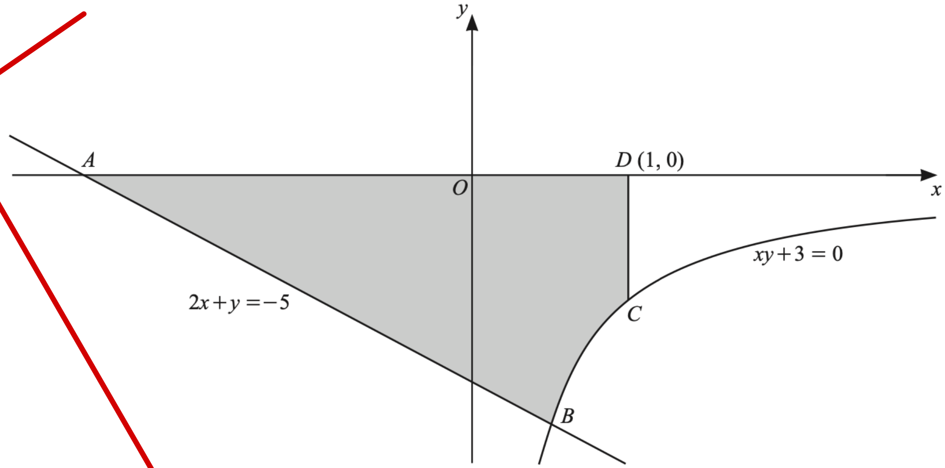
3.



The diagram shows the straight line $2x + y = -5$ and part of the curve $xy + 3 = 0$. The straight line intersects the *x*-axis at the point *A* and intersects the curve at the point *B*. The point *C* lies on the curve. The point *D* has coordinates (1, 0). The line *CD* is parallel to the *y*-axis.

a. Find the coordinates of each of the points *A* and *B*.

[3]

CHAPTER (15)

b. Find the area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are positive integers.

[6]

4. At time ts, a particle travelling in a straight line has acceleration $(2t + 1)^{-\frac{1}{2}} ms^{-2}$. When t = 0, the particle is 4m from a fixed point O and is travelling with velocity $8ms^{-1}$ away from O.

   a. Find the velocity of the particle at time t s. [3]

$$V = 2(2t+1)^{1/2} \times \frac{1}{2} + c$$

$$= (2t+1)^{1/2} + c$$

$$t=0,\ V=8$$

$$8 = 1 + c$$

$$c = 7$$

$$V = (2t+1)^{1/2} + 7$$

   b. Find the displacement of the particle from O at time t s. [4]

$$S = \frac{\cancel{2}}{3}(2t+1)^{3/2} \times \frac{1}{\cancel{2}} + 7t + c$$

$$t=0,\ S=4$$

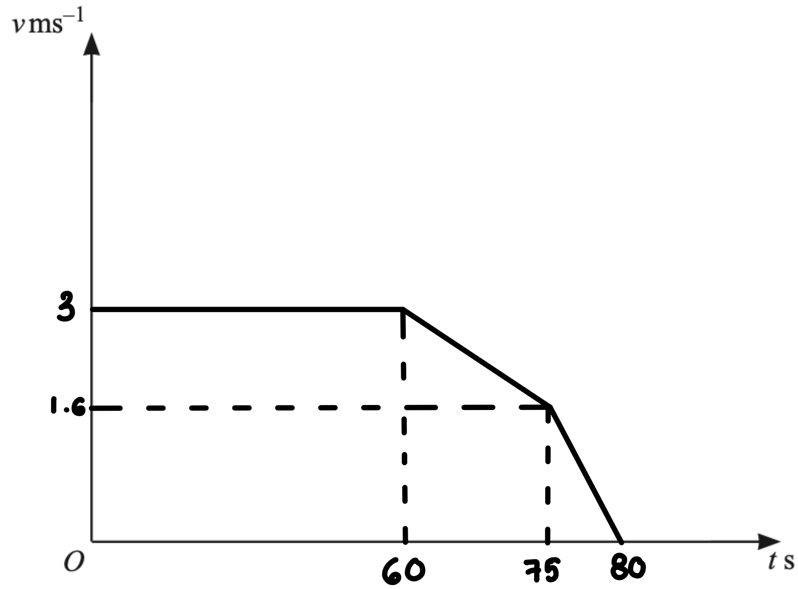
$$4 = \frac{1}{3} + c$$

$$c = \frac{11}{3}$$

$$S = \frac{1}{3}(2t+1)^{3/2} + 7t + \frac{11}{3}$$

5. A particle travels in a straight line. As it passes through a fixed point *O*, the particle is travelling at a velocity of 3 ms^{-1}. The particle continues at this velocity for 60 seconds then decelerates at a constant rate for 15 seconds to a velocity of 1.6 ms^{-1}. The particle then decelerates again at a constant rate for 5 seconds to reach point *A*, where it stops.

   a. Sketch the velocity-time graph for this journey on the axes below. [3]



   b. Find the distance between *O* and *A*. [3]

$$\text{Area} = 60 \times 3 + \frac{1}{2} \times (3 + 1.6) \times 15 + \frac{1}{2} \times 5 \times 1.6$$

$$= 218.5\ m$$

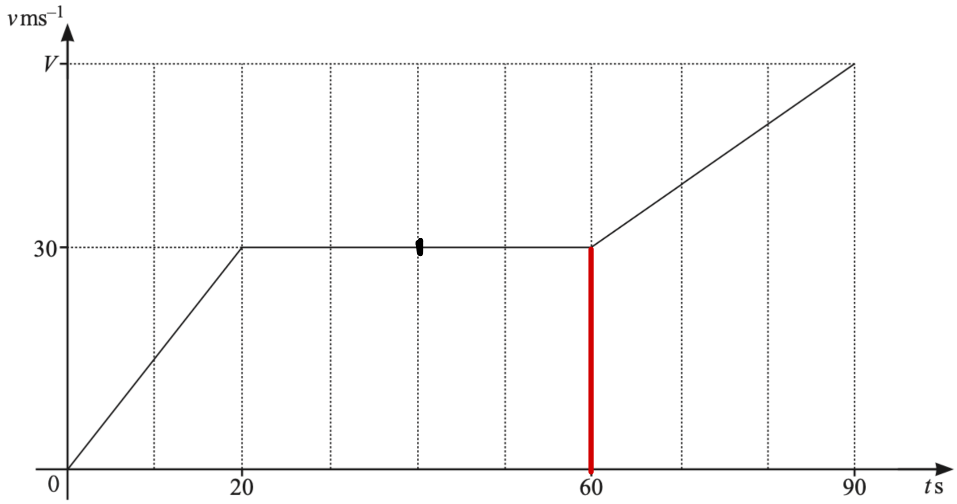
   c. Find the deceleration in the last 5 seconds. [1]

$(80, 0)$ $(75, 1.6)$

$$m = \frac{1.6 - 0}{75 - 80}$$

$$= -0.32$$ deceleration $= 0.32\ ms^{-2}$

6. (a)



The diagram shows the velocity–time graph of a particle P that travels 2775 m in 90 s, reaching a final velocity of $V\,ms^{-1}$

i. Find the value of V. [3]

$A = \frac{1}{\cancel{2}} \times (60+40) \times \cancel{30}^{15} + \frac{1}{\cancel{2}}(30+V) \times \cancel{30}^{15}$

$2775 = 1500 + (30+V)15$

$\frac{1275}{15} = 30+V$

$85 = 30+V$

$V = 55\ ms^{-1}$

ii. Write down the acceleration of P when t = 40. [1]

0 (no slope)

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(b) The acceleration, $a\ ms^{-2}$, of a particle Q travelling in a straight line, is given by $a = 6\cos 2t$ at time t s. When t = 0 the particle is at point O and is travelling with a velocity of 10 ms^{-1}.

I. Find the velocity of Q at time t. [3]

$$v = \frac{6}{2}\sin 2t + C$$

$$v = 3\sin 2t + C$$

$$10 = 3\sin 0 + C$$

$$C = 10$$

$$v = 3\sin 2t + 10$$

II. Find the displacement of Q from O at time t. [3]

$$s = -\frac{3}{2}\cos 2t + 10t + C$$

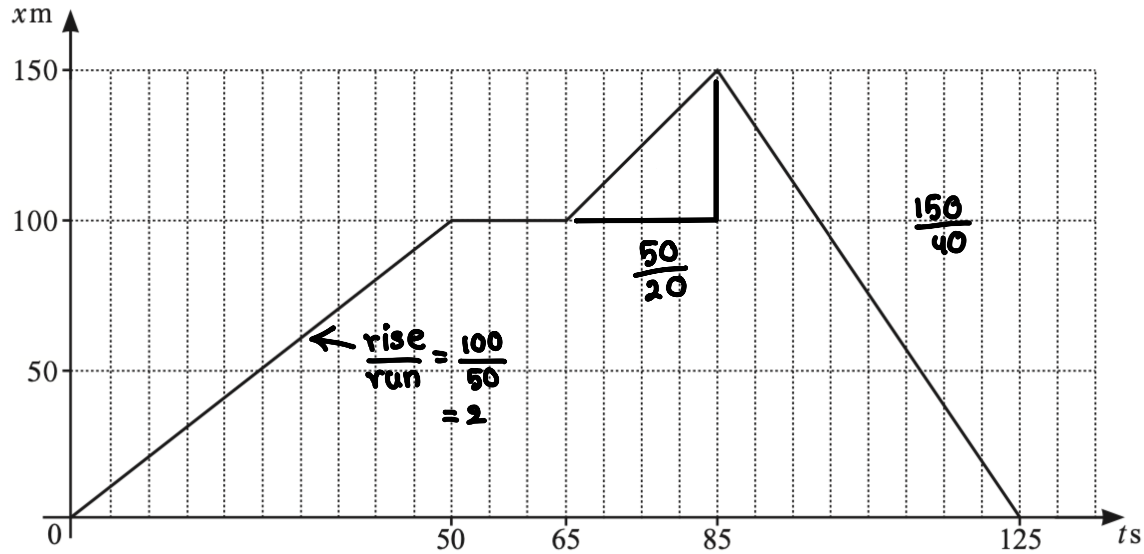
$t=0,\ s=0$

$$0 = -\frac{3}{2} + 0 + C$$

$$C = \frac{3}{2}$$

$$s = -\frac{3}{2}\cos 2t + 10t + \frac{3}{2}$$

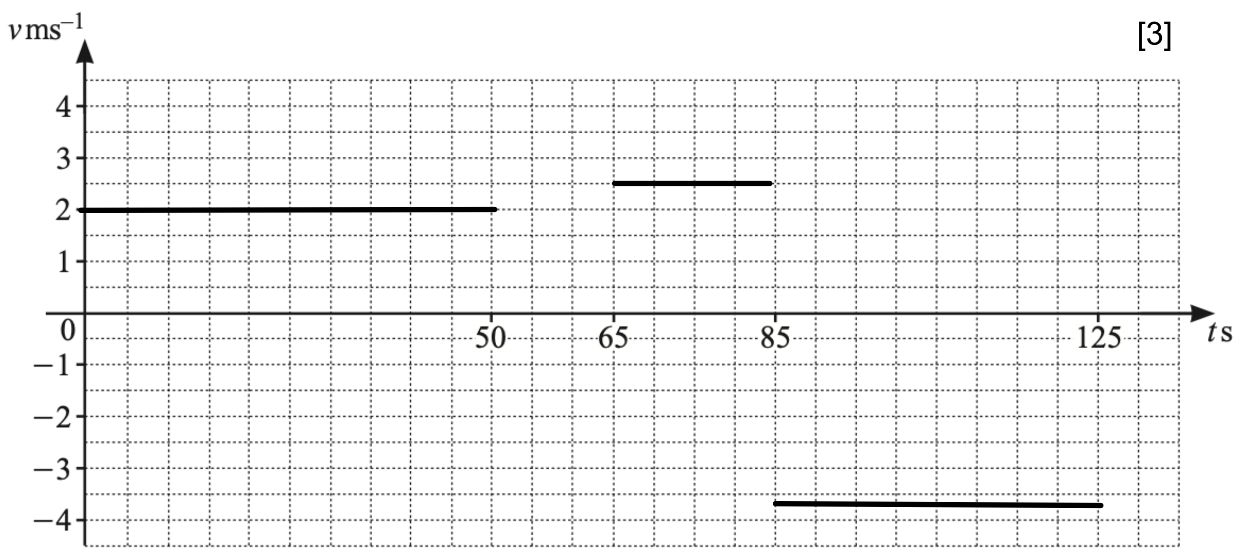
7. (a)



The diagram shows the x–t graph for a runner, where displacement, x, is measured in metres and time, t, is measured in seconds.

(i) On the axes below, draw the v–t graph for the runner.

[3]



(ii) Find the total distance covered by the runner in 125 s.

[1]

$100 + 50 + 150$

$= 300$ m

(b) The displacement, *x* m, of a particle from a fixed point at time *t* s is given by $x = 6\cos(3t + \frac{\pi}{3})$. Find the acceleration of the particle when $t = \frac{2\pi}{3}$.

[3]

$$V = \frac{dx}{dt} = -6\sin(3t + \frac{\pi}{3}) \times 3$$

$$= -18\sin(3t + \frac{\pi}{3})$$

$$a = \frac{dv}{dt} = -54\cos(3t + \frac{\pi}{3})$$

$$t = \frac{2\pi}{3}, \quad a = -54\cos(2\pi + \frac{\pi}{3})$$

$$= -27$$

8. A particle moves in a straight line such that, *t* seconds after passing a fixed point *O*, its displacement from *O* is *s*m, where $s = e^{2t} - 10e^{t} - 12t + 9$.
   a. Find expressions for the velocity and acceleration at time *t*.

[3]

$$V = \frac{ds}{dt} = 2e^{2t} - 10e^{t} - 12$$

$$a = \frac{dv}{dt} = 4e^{2t} - 10e^{t}$$

   b. Find the time when the particle is instantaneously at rest.

$V=0$

[3]

$$2e^{2t} - 10e^{t} - 12 = 0$$

$$e^{2t} - 5e^{t} - 6 = 0$$

$$(e^{t} - 6)(e^{t} + 1) = 0$$

$e^{t} = 6$ $\qquad$ $e^{t} = -1$ (reject)

$t = \ln 6$

c. Find the acceleration at this time.

[2]

$a = 4e^{2t} - 10e^{t}$

$t = \ln 6$

$a = 4e^{2\ln 6} - 10e^{\ln 6}$

$= 4 \times 36 - 10 \times 6$

$= 144 - 60$

$= 84$